



# JENN

**Training and Consultancy**

**The path to enlightened education**

**SUBJECT: SUBJECT NAME**

**GRADE 12**

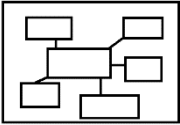



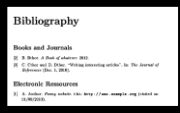
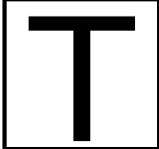
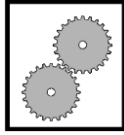

**2025 SPRING**

**SOLUTIONS MANUAL**

**TOPICS**

- 1. FINANCE, GROWTH AND DECAY**
- 2. STATISTICS AND REGRESSION**
- 3. ANALYTICAL GEOMETRY**

## ICON DESCRIPTION

 <p><b>MIND MAP</b></p>	 <p><b>EXAMINATION GUIDELINE</b></p>	 <p><b>CONTENTS</b></p>	 <p><b>ACTIVITIES</b></p>
 <p><b>BIBLIOGRAPHY</b></p>	 <p><b>TERMINOLOGY</b></p>	 <p><b>WORKED EXAMPLES</b></p>	 <p><b>STEPS</b></p>



## **CONTENTS**

## **PAGE**

<b>SECTION 1: Finance, Growth and Decay Solutions</b>	<b>4 – 6</b>
<b>SECTION 2: Statistics and Regression Solutions</b>	<b>7 – 13</b>
<b>SECTION 3: Analytical Geometry Solutions</b>	<b>14 – 28</b>

## Finance, Growth and Decay

May/June 2024

### QUESTION 7/VRAAG 7

7.1	$A = P(1-i)^n$ $8\,337,75 = 13\,000(1-i)^6$ $i = 7,14\%$	✓ substitution in correct formula ✓✓ answer (3)
7.2	$F = \frac{x[(1+i)^n - 1]}{i}$ $80\,000 = \frac{x\left[\left(1 + \frac{8,6}{1200}\right)^{36} - 1\right]}{\frac{8,6}{1200}}$ $x = R1\,955,78$  Thandi's total = $1955,78 \times 36 = R\,70\,408,08$ Eric's total = $1402,31 \times 48 = R\,67\,310,88$ Difference = $70\,409,08 - 67\,310,88$ = $R3\,097,20$	✓ $i$  ✓ substitution into correct formula  ✓ answer   ✓ answer (4)
7.3	$A = P(1+i)^n$ $A = 225\,000\left(1 + \frac{0,09}{12}\right)^3$ $A = R\,230\,100,5637...$  $225\,000\left(1 + \frac{0,09}{12}\right)^3 = \frac{5\,500\left[1 - \left(1 + \frac{0,09}{12}\right)^{-n}\right]}{\frac{0,09}{12}}$ $0,3137734959... = 1 - \left(1 + \frac{0,09}{12}\right)^{-n}$ $\left(1 + \frac{0,09}{12}\right)^{-n} = 0,6862265041...$ $-n = \log_{\left(1 + \frac{0,09}{12}\right)} 0,6862265041...$ $n = 50,394375...$ $n = 51$	✓ substitution in correct formula ✓ answer     ✓ substitution   ✓ simplification  ✓ use of logs  ✓ answer (6)
		[13]

May/June 2023

**QUESTION 6/VRAAG 6**

6.1.1	$A = P(1+i)^n$ $A = 150\,000(1+0,065)^5$ $A = R205\,513$	✓ substitution into the correct formula ✓ answer (2)
6.1.2	$A = P(1-in)$ $A = 150\,000(1-0,09 \times 5)$ $A = 150\,000 - 67\,500$ $A = R82\,500$	✓ substitution into the correct formula ✓ answer (2)
6.1.3	$SF = A - T = 205\,513 - 82\,500$ $= R123\,013$ $F = \frac{x[(1+i)^n - 1]}{i}$ $x = \frac{F \times i}{(1+i)^n - 1}$ $x = \frac{123\,013 \times \frac{0,0785}{12}}{\left[ \left(1 + \frac{0,0785}{12}\right)^{59} - 1 \right] \left(1 + \frac{0,0785}{12}\right)}$ $= R1\,704,01$	✓ answer       ✓ $i = \frac{0,0785}{12}$ ✓ 59 and $\left(1 + \frac{0,0785}{12}\right)$ (A)  ✓ answer (A) (4)
6.2	$P = \frac{x[1 - (1+i)^{-n}]}{i}$ $200\,000 = \frac{6\,000 \left[ 1 - \left(1 + \frac{0,0525}{4}\right)^{-4n} \right]}{\frac{0,0525}{4}}$ $\frac{7}{16} = 1 - \left(1 + \frac{0,0525}{4}\right)^{-4n}$ $\frac{9}{16} = \left(\frac{1621}{1600}\right)^{-4n}$ $-4n = \frac{\log \frac{9}{16}}{\log \left(\frac{1621}{1600}\right)}$ $-4n = -44,1243\dots$ $n = 11,03 \text{ years}$	✓ substitution into correct formula   ✓ simplification    ✓ use of logs  ✓ $-4n = -44,1243\dots$ ✓ $n = 11,03 \text{ years}$ (5)
		<b>[13]</b>

May/June 2022

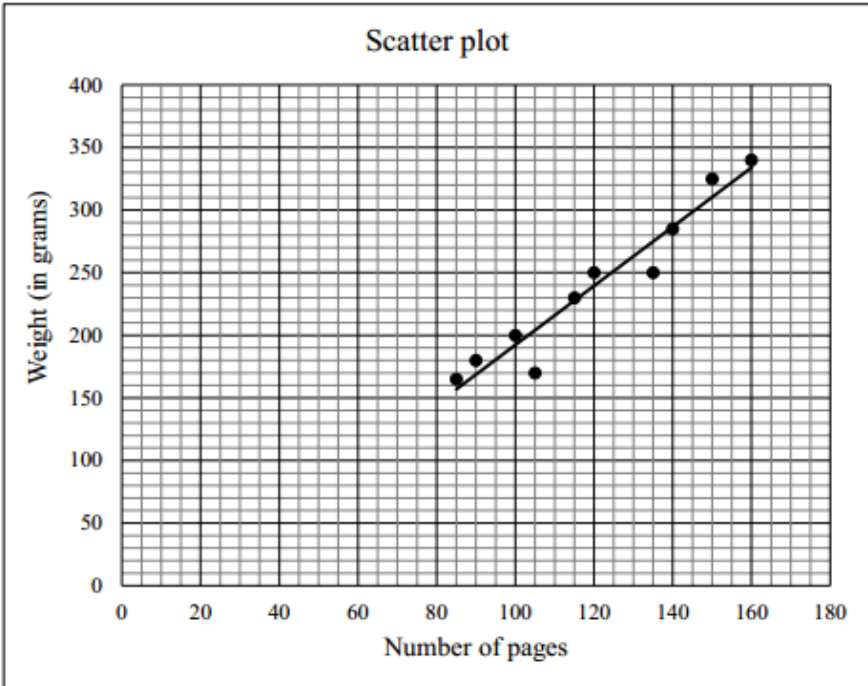
QUESTION 7

7.1	$A = P(1+i)^n$ $2 = 1\left(1 + \frac{0,085}{4}\right)^{4n}$ $4n = \log_{\left(1 + \frac{0,085}{4}\right)} 2$ $n = 8,24 \text{ years}$	$\left. \begin{array}{l} \checkmark 2 \\ \checkmark \frac{0,085}{4} \end{array} \right\}$ In correct formula $\checkmark$ use of logs $\checkmark$ answer in years (4)
7.2.1	$A = P(1-i)^n$ $180\,000 = 500\,000(1-i)^5$ $\frac{9}{25} = (1-i)^5$ $\sqrt[5]{\frac{9}{25}} = 1-i$ $i = 0,1848068\dots$ $r = 18,48\%$	$\checkmark$ subs into correct formula $\checkmark$ simplification $\checkmark i = 0,1848\dots$ $\checkmark$ answer (4)
7.2.2	$A = P(1+i)^n$ $A = 500\,000(1+0,063)^5$ $A = R678\,635,11$	$\checkmark$ subs into correct formula $\checkmark$ answer (2)
7.2.3	Sinking Fund = $678\,635,11 - 180\,000$ = R 498 635,11 $498\,635,11 = \frac{x \left[ \left(1 + \frac{0,1025}{12}\right)^{58} - 1 \right] \left(1 + \frac{0,1025}{12}\right)^3}{\frac{0,1025}{12}}$ $x = R6\,510,36$	$\checkmark$ value of sinking fund $\checkmark \frac{0,1025}{12}$ $\checkmark n = 58 \text{ (A)}$ $\checkmark \left(1 + \frac{0,1025}{12}\right)^3$ $\checkmark$ answer (A) (5)
		<b>[15]</b>

## Statistics and Regression

May/June 2024

### QUESTION/VRAAG 1

1.1	$a = -43,72$ $b = 2,36$ $y = -43,72 + 2,36x$	✓ $a = -43,72$ ✓ $b = 2,36$ ✓ equation (3)
1.2	<p style="text-align: center;">Scatter plot</p>  <p>The scatter plot displays 12 data points. A straight line of best fit is drawn through the points, starting at approximately (85, 160) and ending at (160, 340). The y-axis is labeled 'Weight (in grams)' and ranges from 0 to 400 in increments of 50. The x-axis is labeled 'Number of pages' and ranges from 0 to 180 in increments of 20.</p>	✓ any correct two points ✓ straight line joining the points for $x \in [85 ; 160]$ (2)
1.3	$y = -43,72 + 2,36(110)$ $y = 215,88$ <b>OR</b> $y = 215,90$ (calculator)	✓ substitution ✓ answer (2)  ✓✓ answer (2)

1.4	$y = -43,72 + 2,36(130)$ $y = 263,08$ $\text{Percentage increase in weight} = \frac{263,08 - 215,88}{215,88} \times 100$ $= 21,86\%$ <b>OR</b> $y = 263,08$ $\text{Percentage} = \frac{263,08}{215,88} \times 100$ $= 121,86\%$ $\text{Percentage increase in weight} = 121,86 - 100 = 21,86$	✓ y -value ✓ difference between y-values ✓ +ve answer (3) ✓ y -value ✓ difference between % ✓ +ve answer (3)
		[10]

### QUESTION/VRAAG 2

2.1	<table border="1"> <thead> <tr> <th>Distance (x km)</th><th>Frequency</th><th>Cumulative frequency</th></tr> </thead> <tbody> <tr> <td><math>0 \leq x &lt; 5</math></td><td>3</td><td>3</td></tr> <tr> <td><math>5 \leq x &lt; 10</math></td><td>7</td><td>10</td></tr> <tr> <td><math>10 \leq x &lt; 15</math></td><td>20</td><td>30</td></tr> <tr> <td><math>15 \leq x &lt; 20</math></td><td>12</td><td>42</td></tr> <tr> <td><math>20 \leq x &lt; 25</math></td><td>5</td><td>47</td></tr> <tr> <td><math>25 \leq x &lt; 30</math></td><td>3</td><td>50</td></tr> </tbody> </table>	Distance (x km)	Frequency	Cumulative frequency	$0 \leq x < 5$	3	3	$5 \leq x < 10$	7	10	$10 \leq x < 15$	20	30	$15 \leq x < 20$	12	42	$20 \leq x < 25$	5	47	$25 \leq x < 30$	3	50	✓ 10 ✓ all values correct (2)
Distance (x km)	Frequency	Cumulative frequency																					
$0 \leq x < 5$	3	3																					
$5 \leq x < 10$	7	10																					
$10 \leq x < 15$	20	30																					
$15 \leq x < 20$	12	42																					
$20 \leq x < 25$	5	47																					
$25 \leq x < 30$	3	50																					
2.2	<p style="text-align: center;"><i>Ogive/Ogief</i></p>	✓ grounding ✓ plotting a min of 3 points (cf at upper limits) ✓ smooth, increasing curve (3)																					



2.3	$Q_3 = 17,8$ $Q_1 = 11$  $IQR = 6,8$	✓ $Q_3$ (accept between 17-19) and $Q_1$ (accept between 10-12,5) ✓ answer (accept 5-9) (2)
2.4	$5 \leq x < 10$	✓ $5 \leq x < 10$ (1)
2.5	Estimated mean = $\frac{2,5(3) + 7,5(11) + 12,5(20) + 17,5(8) + 22,5(5) + 27,5(3)}{50}$  $= \frac{675}{50}$  $= 13,5 \text{ km}$	✓ new frequencies  ✓ $\sum fx$  ✓ answer (3)
		[11]

May/June 2023

### QUESTION/VRAAG 1

1.1.1	$a = 1730,22$ $b = 13,96$ $\hat{y} = 1730,22 + 13,96x$	$\checkmark a = 1730,22$ $\checkmark b = 13,96$ $\checkmark$ equation (3)
1.1.2	$\hat{y} = 1730,22 + 13,96x$ $\hat{y} = 1730,22 + 13,96(28\,500)$ $\hat{y} = R399\,590,22$  <b>OR/OF</b>  $\hat{y} = R399\,599,64$ (calc)	$\checkmark$ substitution $\checkmark$ answer (2)  $\checkmark\checkmark$ answer (2)
1.1.3	$r = 0,98002 \dots$ $r = 0,98$	$\checkmark$ answer (1)
1.1.4	There is a very strong positive correlation between the amount spent on advertising and sales. / <i>Daar is 'n baie sterk positiewe korrelasie tussen die bedrag spandeer op advertensie en die verkope.</i>	$\checkmark$ strong positive (1)
1.2.1	$\bar{x} = \frac{1\,552\,195}{9}$ $\bar{x} = 172\,466,11$	$\checkmark \bar{x} = \frac{1\,552\,195}{9}$ $\checkmark$ answer (2)
1.2.2	$\sigma = 56\,950,09$	$\checkmark$ answer (1)
1.2.3	$\bar{x} + \sigma$ $= 172\,466,11 + 56\,950,09$ $= 229\,416,20$  2 years/jaar	$\checkmark \bar{x} + \sigma$ $\checkmark$ answer (2)
		<b>[12]</b>

QUESTION/VRAAG 2

2.1	$35 < x \leq 45$	✓ answer (1)																								
2.2	320 people/mense	✓ answer (1)																								
2.3	<table border="1"> <thead> <tr> <th>AGE</th><th>NUMBER OF PEOPLE</th><th>CUMULATIVE FREQUENCY</th></tr> </thead> <tbody> <tr> <td><math>5 &lt; x \leq 15</math></td><td>20</td><td>20</td></tr> <tr> <td><math>15 &lt; x \leq 25</math></td><td>25</td><td>45</td></tr> <tr> <td><math>25 &lt; x \leq 35</math></td><td>60</td><td>105</td></tr> <tr> <td><math>35 &lt; x \leq 45</math></td><td>90</td><td>195</td></tr> <tr> <td><math>45 &lt; x \leq 55</math></td><td>55</td><td>250</td></tr> <tr> <td><math>55 &lt; x \leq 65</math></td><td>40</td><td>290</td></tr> <tr> <td><math>65 &lt; x \leq 75</math></td><td>30</td><td>320</td></tr> </tbody> </table>	AGE	NUMBER OF PEOPLE	CUMULATIVE FREQUENCY	$5 < x \leq 15$	20	20	$15 < x \leq 25$	25	45	$25 < x \leq 35$	60	105	$35 < x \leq 45$	90	195	$45 < x \leq 55$	55	250	$55 < x \leq 65$	40	290	$65 < x \leq 75$	30	320	
AGE	NUMBER OF PEOPLE	CUMULATIVE FREQUENCY																								
$5 < x \leq 15$	20	20																								
$15 < x \leq 25$	25	45																								
$25 < x \leq 35$	60	105																								
$35 < x \leq 45$	90	195																								
$45 < x \leq 55$	55	250																								
$55 < x \leq 65$	40	290																								
$65 < x \leq 75$	30	320																								
	<p style="text-align: center;"><b>OGIVE/OGIEF</b></p>	<ul style="list-style-type: none"> <li>✓ cumulative frequency</li> <li>✓ grounding</li> <li>✓ plotting at upper limit</li> <li>✓ shape</li> </ul> <p style="text-align: right;">(4)</p>																								
2.4	Median = 41	✓✓ answer (2)																								
		[8]																								

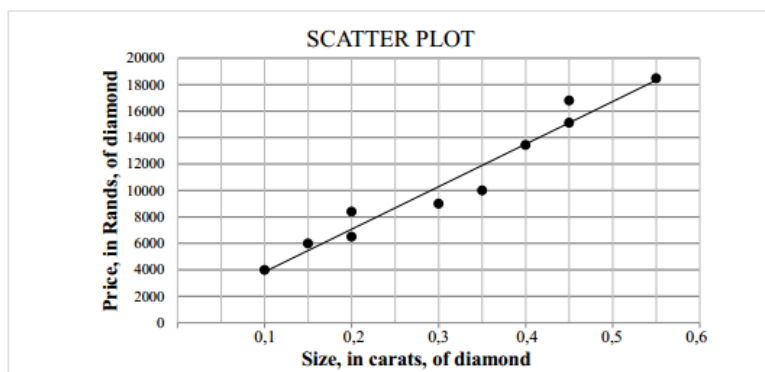
## QUESTION/VRAAG 1

1.1	Modal class: $9 < m \leq 11$	✓ answer (1)																								
1.2	<table border="1"> <thead> <tr> <th>Mass (in kg)</th><th>Frequency</th><th>Cumulative frequency</th></tr> </thead> <tbody> <tr> <td><math>5 &lt; m \leq 7</math></td><td>6</td><td>6</td></tr> <tr> <td><math>7 &lt; m \leq 9</math></td><td>18</td><td>24</td></tr> <tr> <td><math>9 &lt; m \leq 11</math></td><td>21</td><td>45</td></tr> <tr> <td><math>11 &lt; m \leq 13</math></td><td>19</td><td>64</td></tr> <tr> <td><math>13 &lt; m \leq 15</math></td><td>11</td><td>75</td></tr> <tr> <td><math>15 &lt; m \leq 17</math></td><td>4</td><td>79</td></tr> <tr> <td><math>17 &lt; m \leq 19</math></td><td>1</td><td>80</td></tr> </tbody> </table>	Mass (in kg)	Frequency	Cumulative frequency	$5 < m \leq 7$	6	6	$7 < m \leq 9$	18	24	$9 < m \leq 11$	21	45	$11 < m \leq 13$	19	64	$13 < m \leq 15$	11	75	$15 < m \leq 17$	4	79	$17 < m \leq 19$	1	80	✓ adding       ✓ 80 (2)
Mass (in kg)	Frequency	Cumulative frequency																								
$5 < m \leq 7$	6	6																								
$7 < m \leq 9$	18	24																								
$9 < m \leq 11$	21	45																								
$11 < m \leq 13$	19	64																								
$13 < m \leq 15$	11	75																								
$15 < m \leq 17$	4	79																								
$17 < m \leq 19$	1	80																								
1.3		✓ grounding (5 ; 0)  ✓ points  ✓ shape (3)																								
1.4	Median mass: 10,5 kg	✓✓ answer (2)																								
1.5.1	$\bar{x} = \frac{(6 \times 6 + 18 \times 8 + 21 \times 10 + 19 \times 12 + 11 \times 14 + 4 \times 16 + 1 \times 18)}{80}$ $= \frac{854}{80}$ $= 10,68$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">Answer only 2/2</div>	✓ 854  ✓ answer (2)																								
1.5.2	Learners' bags are heavier than the stipulated international guideline. Estimated mean = 10,68 kg 10% of 80 kg = 8 kg 10,68 kg > 8 kg	✓ answer    ✓ 8 kg (2)																								

<p><b>OR/ OF</b></p> <p>Learners' bags are heavier than the stipulated international guideline.</p> <p>Estimated mean <math>= \frac{10,68}{80} \times 100</math></p> <p><math>= 13,35\%</math></p> <p><math>13,35\% &gt; 10\%</math></p>	<p>✓ answer</p> <p>✓ 13,35%</p> <p>(2)</p>
<b>[12]</b>	

#### QUESTION/VRAAG 2

Size, in carats, of diamond (x)	0,1	0,15	0,2	0,2	0,3	0,35	0,4	0,45	0,45	0,55
Price, in rands, of diamond (y)	4 000	6 000	6 500	8 400	9 000	10 000	13 440	15 120	16 800	18 480



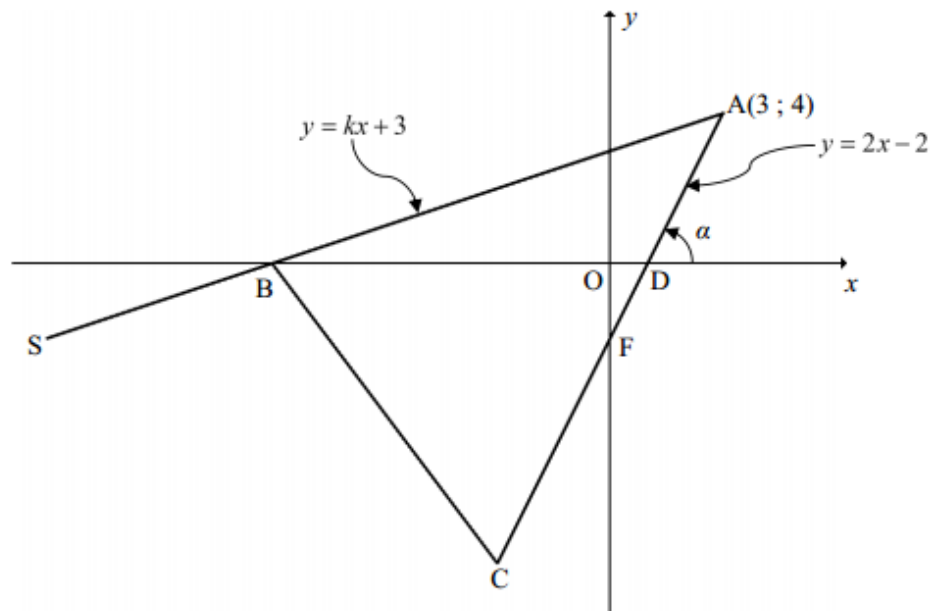
2.1	$a = 634,382...$ $b = 32\,189,263...$ $\hat{y} = 634,38 + 32189,26x$	<p>Answer only 3/3</p> <p>✓ a ✓ b ✓ equation</p>	(3)
2.2	$\hat{y} = 634,38 + 32189,26(0,25)$ $= R8\,681,70$ <b>OR/OF</b> $\hat{y} = R8\,681,70$ (if using calculator)	<p>✓ substitution ✓ answer</p> <p>✓ ✓ answer</p>	(2) (2)
2.3	<p>Average price increase <math>= R \frac{32189,26}{20}</math> per 0,05 carat  <math>= R1\,609,46</math> per 0,05 carat</p> <p><b>OR/OF</b></p> <p>Average price increase <math>= 0,05 \times 32189,26</math>  <math>= R1\,609,46</math> per 0,05 carat</p> <p><b>OR/OF</b></p> <p>at 0,3: <math>\hat{y} = R10\,291,16</math>  <math>\therefore</math> Average price increase <math>= 10\,291,16 - 8\,681,70</math>  <math>= R1\,609,46</math> per 0,05 carat</p> <p>Answer only 2/2</p>	<p>✓ divide gradient by 20 ✓ answer</p> <p>✓ multiply gradient by 0,05 ✓ answer</p> <p>✓ Estimated price of a 0,3 carat diamond ✓ answer</p>	(2) (2) (2)
2.4	The point (0,35 ; 11500) is closer to the least squares regression line.	✓ reason	(1)

**[8]**

## Analytical Geometry

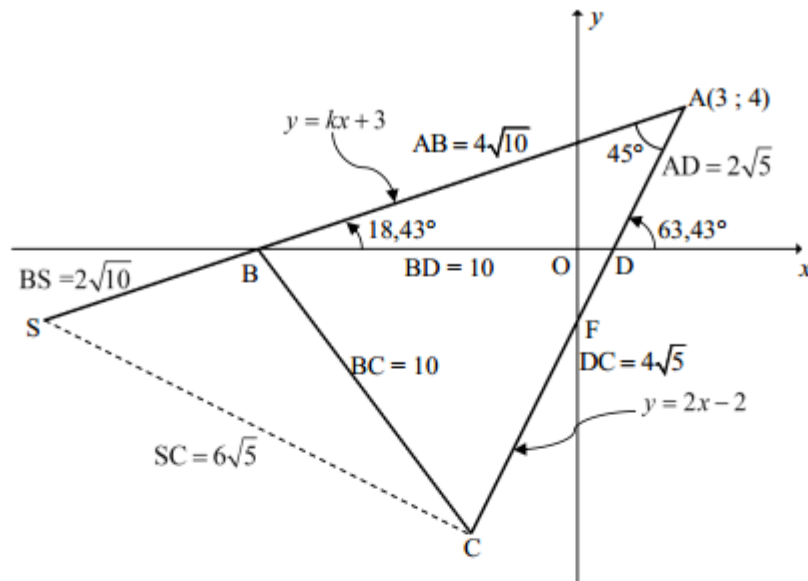
May/June 2024

### QUESTION/VRAAG 3



3.1	$y = kx + 3$ $4 = k(3) + 3$ $3k = 1$ $\therefore k = \frac{1}{3}$  <b>OR</b> y-intercept of AB: (0 ; 3)  $m_{AB} = \frac{4-3}{3-0}$ $= \frac{1}{3}$ $\therefore k = \frac{1}{3}$	✓ substitution (3 ; 4)          ✓ substitution (0 ; 3)	(1)
3.2	$0 = \frac{1}{3}x + 3$ $-3 = \frac{1}{3}x$ $x = -9$ B(-9 ; 0)	✓ $y = 0$    ✓ answer	(2)

3.3	$F(0; -2)$ $F\left(\frac{x+3}{2}; \frac{y+4}{2}\right)$ $\frac{x+3}{2} = 0 \quad \frac{y+4}{2} = -2$ $x = -3 \quad y = -8$ $C(-3; -8)$  <b>OR</b> by translation  $F(0; -2)$ $A \rightarrow F(x; y) \rightarrow (x-3; y-6)$ $F \rightarrow C(0; -2) \rightarrow (0-3; -2-6) = (-3; -8)$	$\checkmark F(0; -2)$  $\checkmark \frac{x+3}{2} = 0; \frac{y+4}{2} = -2$  $\checkmark x\text{-value} \quad \checkmark y\text{-value}$ (4)  $\checkmark F(0; -2)$ $\checkmark (x-3; y-6)$ $\checkmark x\text{-value} \quad \checkmark y\text{-value}$ (4)
3.4	$m_{BC} = \frac{0 - (-8)}{-9 - (-3)} \quad \text{OR} \quad m_{BC} = \frac{-8 - 0}{-3 - (-9)}$  $m_{BC} = -\frac{4}{3}$ $y = -\frac{4}{3}x + c$ $(-2) = -\frac{4}{3}(-15) + c$  $c = -22$ $y = -\frac{4}{3}x - 22$  <b>OR</b>  $m_{BC} = \frac{0 - (-8)}{-9 - (-3)} \quad \text{OR} \quad m_{BC} = \frac{-8 - 0}{-3 - (-9)}$  $m_{BC} = -\frac{4}{3}$  $y - y_1 = -\frac{4}{3}(x - x_1)$ $y - (-2) = -\frac{4}{3}(x - (-15))$ $y + 2 = -\frac{4}{3}x - 20$ $y = -\frac{4}{3}x - 22$	$\checkmark$ substitution of B and C into the gradient formula  $\checkmark m_{BC}$  $\checkmark m_{\text{line}} = m_{BC}$ $\checkmark$ substitution of $S(-15; -2)$  $\checkmark$ equation (5)  $\checkmark$ substitution into the gradient formula  $\checkmark m_{BC}$  $\checkmark m_{\text{line}} = m_{BC}$ $\checkmark$ substitution of $S(-15; -2)$  $\checkmark$ equation (5)

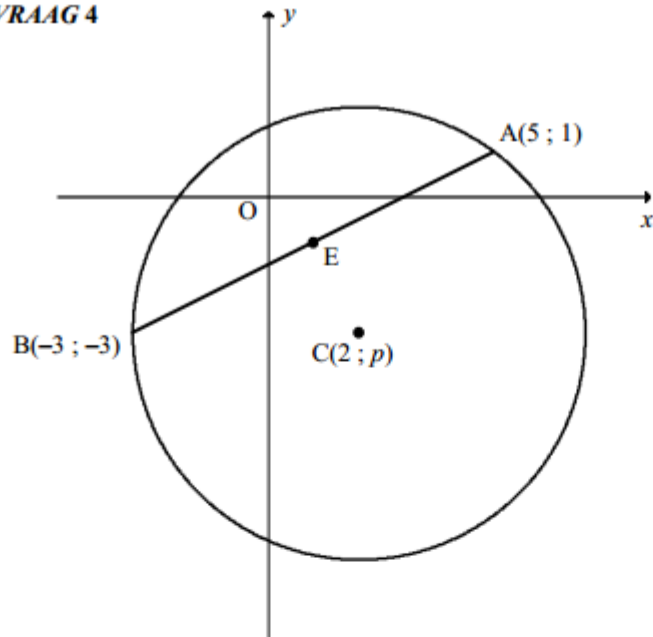


3.5	<p> <math>\tan \alpha = m_{AC} = 2</math>  <math>\alpha = 63,43^\circ</math>  <math>\tan \hat{ABD} = m_{AS} = \frac{1}{3}</math>  <math>\hat{ABD} = 18,43^\circ</math>  <math>\hat{BAC} = \alpha - \hat{ABD}</math>  <math>\hat{BAC} = 63,43^\circ - 18,43^\circ</math>  <math>\hat{BAC} = 45^\circ</math> </p> <p><b>OR</b></p> <p> <math>AB = \sqrt{(-9-3)^2 + (0-4)^2}</math>  <math>AB = 4\sqrt{10}</math>  <math>BD = 10</math>  <math>AD = \sqrt{(3-1)^2 + (4-0)^2}</math>  <math>AD = 2\sqrt{5}</math> </p> <p> <math>BD^2 = AB^2 + AD^2 - 2AB \cdot AD \cos \hat{BAC}</math>  <math>(10)^2 = (4\sqrt{10})^2 + (2\sqrt{5})^2 - 2(4\sqrt{10})(2\sqrt{5}) \cos \hat{BAC}</math>  <math>\cos \hat{BAC} = \frac{\sqrt{2}}{2}</math>  <math>\hat{BAC} = 45^\circ</math> </p>	<p> <math>\checkmark \tan \alpha = m_{AC} = 2</math>  <math>\checkmark \alpha = 63,43^\circ</math>  <math>\checkmark \tan \hat{ABD} = m_{AS} = \frac{1}{3}</math>  <math>\checkmark \hat{ABD} = 18,43^\circ</math> </p> <p><math>\checkmark</math> answer (5)</p> <p><math>\checkmark</math> length of AB</p> <p><math>\checkmark</math> calculation of remaining 2 lengths</p> <p><math>\checkmark</math> substitution into cosine-rule</p> <p><math>\checkmark</math> rewriting in terms of <math>\cos \hat{BAC}</math></p> <p><math>\checkmark</math> answer (5)</p>
-----	--	---






QUESTION/VRAAG 4

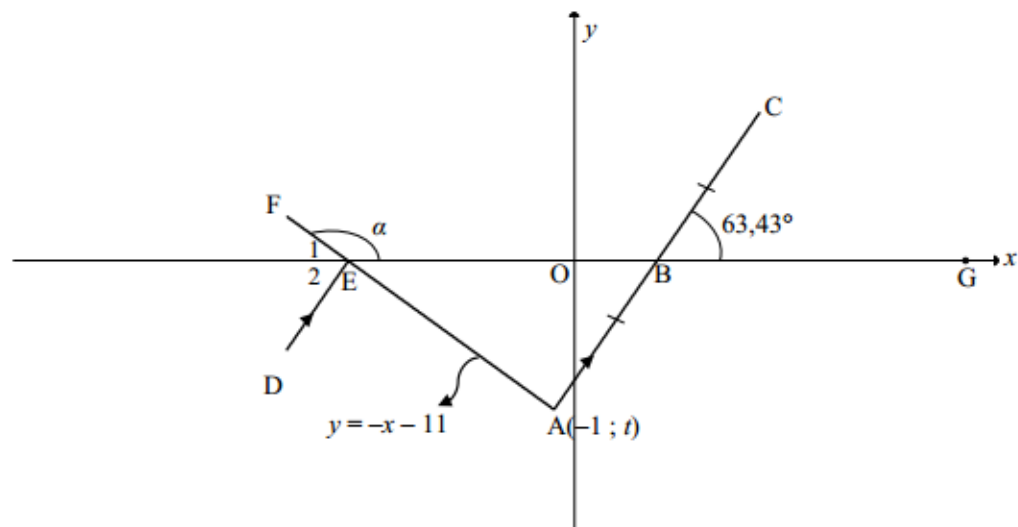


4.1	$E\left(\frac{5+(-3)}{2}; \frac{1+(-3)}{2}\right)$ $\therefore E(1; -1)$	$\checkmark x=1 \quad \checkmark y=-1$ (2)
4.2	$AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$ $AB = \sqrt{(5 - (-3))^2 + (1 - (-3))^2}$ $AB = \sqrt{80} = 4\sqrt{5} = 8,94 \text{ units}$	$\checkmark AB = \sqrt{80} = 4\sqrt{5} = 8,94$ (1)
4.3	$m_{AB} = \frac{1 - (-3)}{5 - (-3)}$ $m_{AB} = \frac{1}{2}$ $\therefore m_{CE} = -2 \quad [CE \perp AB]$ $E(1; -1)$ $y = -2x + c \quad \text{OR} \quad y - y_1 = -2(x - x_1)$ $(-1) = -2(1) + c \quad y - (-1) = -2(x - 1)$ $c = 1 \quad y = -2x + 1$	$\checkmark m_{AB} = \frac{1}{2}$ $\checkmark m_{CE}$ $\checkmark$ substitution of E $\checkmark$ equation (4)

4.4	$y = -2x + 1$ $p = -2(2) + 1$ $p = -3$  <b>OR</b>  $m_{CE} = -2$ $\frac{p - (-1)}{2 - 1} = -2$ $p + 1 = -2$ $p = -3$	✓ substitution of C(2 ; p) into $\perp$ bisector of AB  (1)   ✓ substitution of C and E into the gradient formula  (1)
4.5	$BC = r = 5$ units  $\therefore (x - 2)^2 + (y + 3)^2 = 25$ $x^2 - 4x + 4 + y^2 + 6y + 9 = 25$ $x^2 + y^2 - 4x + 6y - 12 = 0$	✓ $BC = r = 5$ units  ✓ $(x - 2)^2 + (y + 3)^2 = r^2$ ✓ $x^2 - 4x + 4 + y^2 + 6y + 9 = 25$  (4)

4.6	$(x - 2)^2 + (y + 3)^2 = 25$ $y = tx + 8$ $(x - 2)^2 + (tx + 8 + 3)^2 = 25$ $x^2 - 4x + 4 + t^2x^2 + 22tx + 121 - 25 = 0$ $x^2(t^2 + 1) + x(22t - 4) + 100 = 0$  $\Delta < 0$  $(22t - 4)^2 - 4(t^2 + 1)(100) < 0$ $484t^2 - 176t + 16 - 400t^2 - 400 < 0$ $84t^2 - 176t - 384 < 0$ $21t^2 - 44t - 96 < 0$ $(7t - 24)(3t + 4) < 0$  CV: $\frac{24}{7}; -\frac{4}{3}$    $\therefore t \in \left(-\frac{4}{3}; \frac{24}{7}\right)$ <b>OR</b> $-\frac{4}{3} < t < \frac{24}{7}$	✓ substitution of $y = tx + 8$ ✓ standard form ✓ $\Delta < 0$      ✓ standard form of $\Delta$  ✓ critical values     ✓ answer  (6)
		[18]

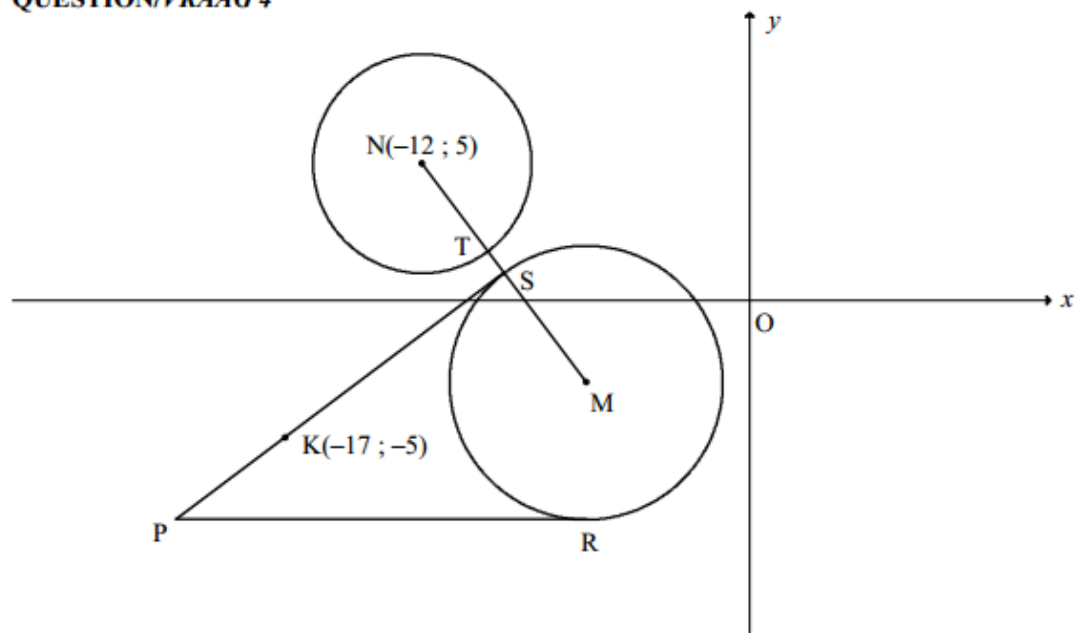
QUESTION/VRAAG 3



3.1.1	$y = -x - 11$ $A(-1 ; t)$ $t = -(-1) - 11$ $t = -10$	✓ substitution ✓ value of $t$ (2)
3.1.2	$\tan \alpha = -1$ $ref. \angle = 45^\circ$ $\therefore \alpha = 135^\circ$	✓ $\tan \alpha = -1$ ✓ $135^\circ$ (2)
3.1.3	$\tan 63,43^\circ = m_{AC}$ $m_{AC} = 2$	✓ $\tan 63,43^\circ = m_{AC}$ ✓ answer (2)
3.2	$m_{AC} = 2$ $A(-1 ; -10)$ $y = 2x + k$ $-10 = 2(-1) + k$ $k = -8$ $y = 2x - 8$  <b>OR/OF</b> $y - y_1 = 2(x - x_1)$ $y - (-10) = 2(x - (-1))$ $y = 2x - 8$	✓ substitution of $m$ and $A$ ✓ equation (2)

3.3.1	$y = 2x - 8$ $0 = 2x - 8$ $x_B = 4$  $\frac{x_C + (-1)}{2} = 4$ $x_C = 9$  $\frac{y_C + (-10)}{2} = 0$ $y_C = 10$  <b>OR/OF</b> by translation / <i>met translasi</i>  $A \rightarrow B(x; y) \rightarrow (x+5; y+10)$ $B \rightarrow C(4; 0) \rightarrow (4+5; 0+10) = (9; 10)$	$\checkmark x_B = 4$  $\checkmark x_C = 9 \quad \checkmark y_C = 10$ (3)  $\checkmark (x+5; y+10)$ $\checkmark x_C = 9 \quad \checkmark y_C = 10$ (3)
3.3.2	$\hat{A}BE = 63,43^\circ$ $\hat{E}_2 = 63,43^\circ$ $\hat{E}_1 = 45^\circ$ $\hat{F}ED = 108,43^\circ$  <b>OR/OF</b>  $\hat{E}AB = 135^\circ - 63,43^\circ$ $\hat{E}AB = 71,57^\circ$ $\hat{D}EA = \hat{E}AB = 71,57^\circ$ $\hat{F}ED = 108,43^\circ$  <b>OR/OF</b>  $\hat{A}BE = 63,43^\circ$ $\hat{D}EO = 116,57^\circ$ $\hat{F}ED = 360^\circ - (116,57^\circ + 135^\circ)$ $= 108,43^\circ$	[vert. opp $\angle$ 's =] [corres. $\angle$ 's, DE $\parallel$ AB] [ $\angle$ s on a str line]  $\checkmark \hat{A}BE = 63,43^\circ$ $\checkmark \hat{E}_1 = 45^\circ$ $\checkmark \hat{F}ED = 108,43^\circ$ (3)  $\checkmark \hat{E}AB = 71,57^\circ$ $\checkmark \hat{D}EA = \hat{E}AB = 71,57^\circ$ $\checkmark \hat{F}ED = 108,43^\circ$ (3)  [vert. opp $\angle$ 's] [co-int. $\angle$ 's, DE $\parallel$ AB] $\checkmark \hat{A}BE = 63,43^\circ$ $\checkmark \hat{D}EO = 116,57^\circ$ $\checkmark \hat{F}ED = 108,43^\circ$ (3)
3.4	$y = 0$ $x_E = -11$ $\frac{x_G + (-11)}{2} = 4$ $x_G = 19$  $(x-19)^2 + y^2 = 15^2$ $(x-19)^2 + y^2 = 225$	$\checkmark x_E = -11$  $\checkmark x_G = 19$  $\checkmark (x-19)^2 + y^2 = 225$ (4)

QUESTION/VRAAG 4



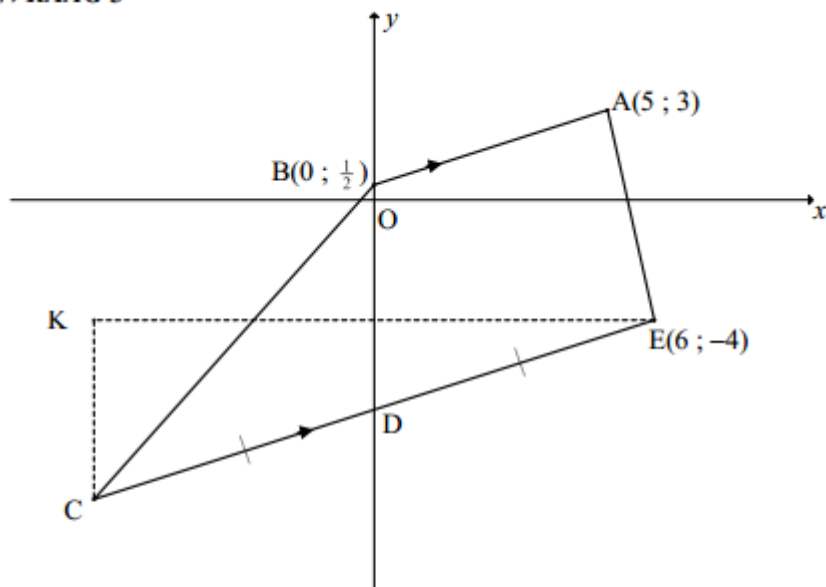
4.1	$M(-6; -3)$	✓ $-6$ ✓ $-3$ (2)
4.2.1	$x^2 + y^2 + 24x - 10y + 153 = 0$ $(x+12)^2 + (y-5)^2 = -153 + 144 + 25$ $(x+12)^2 + (y-5)^2 = 16$ $r^2 = 16$ $r = 4$ units	✓ $r^2 = -153 + 144 + 25$ ✓ length of radius (2)
4.2.2	$NM = \sqrt{(-12 - (-6))^2 + (5 - (-3))^2}$ $NM = 10$ units $SM = 5$ units $\therefore TS = 10 - 5 - 4 = 1$ unit	✓ substitution into distance formula ✓ $NM = 10$ units ✓ $SM = 5$ units ✓ answer (4)
4.3.1	$R(-6; -8)$ $y = -8$	✓ $y_R = -8$ ✓ answer (2)

4.3.2	$m_{NM} = \frac{5 - (-3)}{-12 - (-6)}$ $m_{NM} = -\frac{4}{3}$ $m_{\text{tangent}} = \frac{3}{4}$ $-5 = \frac{3}{4}(-17) + c \quad \text{OR/OF} \quad y - y_1 = \frac{3}{4}(x - x_1)$ $c = \frac{31}{4} \quad y - (-5) = \frac{3}{4}(x - (-17))$ $y = \frac{3}{4}x + \frac{31}{4} \quad y = \frac{3}{4}x + \frac{31}{4}$ <p><b>OR/OF</b></p> $NS = SM = 5$ $S\left(\frac{-12-6}{2}; \frac{5-3}{2}\right)$ $S(-9; 1)$ $m_{SK} = \frac{1 - (-5)}{-9 - 17}$ $= \frac{6}{8} = \frac{3}{4}$ $y + 5 = \frac{3}{4}(x + 17)$ $y = \frac{3}{4}x + \frac{31}{4} \quad \text{or} \quad y = \frac{3}{4}x + 7\frac{3}{4}$	<p>✓ substitution</p> <p>✓ <math>m_{NM} = -\frac{4}{3}</math></p> <p>✓ <math>m_{\text{tangent}} = \frac{3}{4}</math></p> <p>✓ substitution of <math>m</math> and <math>N</math></p> <p>✓ equation (5)</p> <p>✓ <math>S</math> midpoint</p> <p>✓ coordinates of <math>S</math></p> <p>✓ <math>m_{\text{tangent}} = \frac{3}{4}</math></p> <p>✓ substitution of <math>m</math> and <math>K(-17; -5)</math> or <math>S</math></p> <p>✓ equation (5)</p>
4.4.1	$-8 = \frac{3}{4}x + \frac{31}{4}$ $-32 = 3x + 31$ $3x = -63$ $x = -21$ $P(-21; -8)$ $R(-6; -8)$ <p><math>PR = PS = 15</math> units [tangents from same point]</p> <p><math>MS = MR = 5</math> units</p> <p>Perimeter <math>PSMR = 15 + 15 + 5 + 5</math>  <math>= 40</math> units</p>	<p>✓ <math>-8 = \frac{3}{4}x + \frac{31}{4}</math></p> <p>✓ <math>x = -21</math></p> <p>✓ <math>PR = PS = 15</math> units</p> <p>✓ <math>MS = MR = 5</math> units</p> <p>✓ answer (5)</p>

4.4.2	$\frac{\text{area of } \triangle NPS}{\text{area of quadrilateral PSMR}}$ $\frac{\frac{1}{2}NS.SP}{\frac{1}{2}SP.MS + \frac{1}{2}MR.PR}$ $= \frac{\frac{1}{2}(5)(15)}{2\left(\frac{1}{2}\right)(5)(15)}$ $= \frac{1}{2}$ <p><b>OR</b></p> $\triangle NPS \cong \triangle SPM \cong \triangle MPR$ $\frac{\text{area of } \triangle NPS}{\text{area of quadrilateral PSMR}}$ $= \frac{1}{2}$	<p>✓ substitution</p> <p>✓ answer (2)</p> <p>✓ congruent</p> <p>✓ answer (2)</p>
<b>[22]</b>		

May/June 2022

**QUESTION/VRAAG 3**



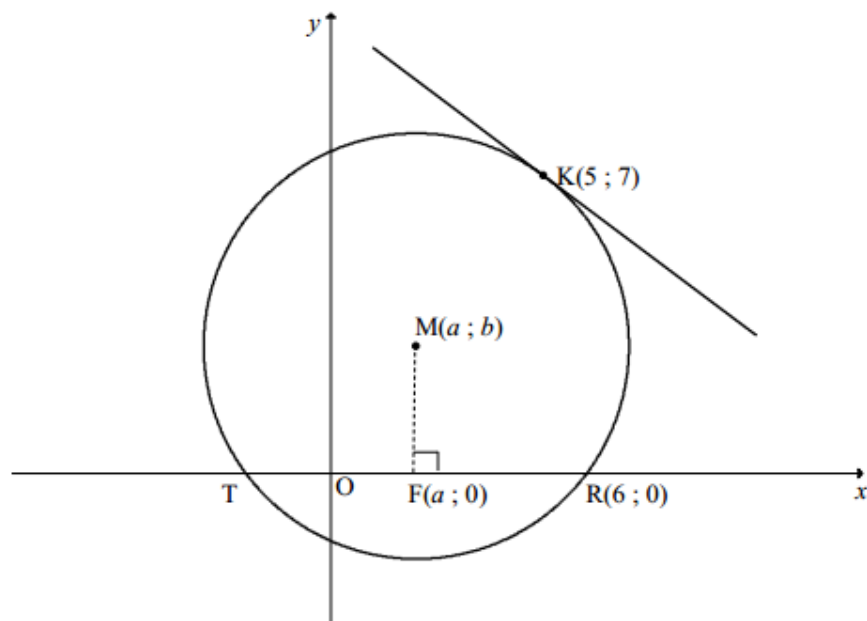
3.1	$m_{AB} = \frac{3 - \frac{1}{2}}{5 - 0}$ $m_{AB} = \frac{1}{2}$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">Answer only 2/2</div>	<p>✓ substitution</p> <p>✓ answer (2)</p>
-----	--	---



3.2	$m_{CE} = m_{BA} = \frac{1}{2}$ $-4 = \frac{1}{2}(6) + c$ <b>OR/OR</b> $y - (-4) = \frac{1}{2}(x - 6)$ $c = -7$ $y = \frac{1}{2}x - 7$	✓ gradient ✓ substitution of E ✓ answer (3)
3.3.1	D(0 ; -7) $\frac{x_c + 6}{2} = 0$ $\frac{y_c + (-4)}{2} = -7$ $x_c = -6$ $y_c = -10$ C(-6 ; -10) <div style="border: 1px solid black; padding: 2px; display: inline-block;">Answer only 3/3</div>	✓ D(0 ; -7) ✓ $x_c = -6$ ✓ $y_c = -10$ (3)
3.3.2	Area $\triangle BCD = \frac{1}{2}(7,5)(6)$ $= 22,5$ Area $\triangle ABD = \frac{1}{2}(7,5)(5)$ $= 18,75$ Area ABCD = $22,5 + 18,75 = 41,25 \text{ units}^2$	✓ subst of correct base and height into the area formula ✓ area $\triangle BCD = 22,5$  ✓ area $\triangle ABD = 18,75$ ✓ answer (4)

3.4.1	$K(-6; -4)$	$\checkmark x_K = -6$ $\checkmark y_K = -4$ (2)
3.4.2a	$KC = 6$ units; $KE = 12$ units; $CE = \sqrt{(6)^2 + (12)^2}$ [Pythagoras] $CE = \sqrt{180} = 6\sqrt{5} = 13,42$ Perimeter $\triangle KEC = 6 + 12 + \sqrt{180}$ $= 31,42$ units	$\checkmark KC = 6$ units $\checkmark KE = 12$ units  $\checkmark CE$  $\checkmark$ answer (4)
3.4.2b	$\tan \hat{KCE} = \frac{KE}{KC} = \frac{12}{6} = 2$ $\hat{KCE} = 63,43^\circ$  <b>OR/OF</b>  $\sin \hat{KCE} = \frac{KE}{CE} = \frac{12}{\sqrt{180}} = \frac{2\sqrt{5}}{5}$ $\hat{KCE} = 63,43^\circ$  <b>OR/OF</b>  $m_{CE} = \frac{1}{2}$ $\tan \theta = \frac{1}{2}$ $\theta = 26,57^\circ$ $\hat{KCE} = 90^\circ - 26,57^\circ$ $\hat{KCE} = 63,43^\circ$  <b>OR/OF</b>  $KE^2 = KC^2 + CE^2 - 2(KC)(CE)\cos \hat{KCE}$ $(12)^2 = (6)^2 + (\sqrt{180})^2 - 2(6)(\sqrt{180})(\cos \hat{KCE})$ $\cos \hat{KCE} = \frac{\sqrt{5}}{5}$ $\hat{KCE} = 63,43^\circ$	$\checkmark$ trig ratio $\checkmark \tan \hat{KCE} = 2$ $\checkmark$ answer (3)   $\checkmark$ trig ratio $\checkmark \sin \hat{KCE} = \frac{12}{\sqrt{180}}$ $\checkmark$ answer (3)   $\checkmark \tan \theta = \frac{1}{2}$ $\checkmark \theta = 26,57^\circ$  $\checkmark$ answer    $\checkmark$ substitution into cosine rule $\checkmark$ trig ratio $\checkmark$ answer (3)

QUESTION/VRAAG 4



4.1.1	$y = x + 1$ $b = a + 1$	$\checkmark b = a + 1$ (1)
4.1.2	$MR^2 = MK^2$ $(a - 6)^2 + (b - 0)^2 = (a - 5)^2 + (b - 7)^2$ $(a - 6)^2 + (a + 1)^2 = (a - 5)^2 + (a + 1 - 7)^2$ $a^2 + 2a + 1 = a^2 - 10a + 25$ $12a = 24$ $a = 2$ $b = 3$ $\therefore M(2; 3)$	$\checkmark$ equating radii / solving simultaneously $\checkmark$ substitution $b = a + 1$ $\checkmark 12a = 24$ $\checkmark a = 2$ $\checkmark b = 3$ (5)
4.2.1	$(6 - 2)^2 + (0 - 3)^2 = r^2$ $r = 5$ <b>OR/OF</b> $(2 - 5)^2 + (3 - 7)^2 = r^2$ $r = 5$	$\checkmark$ substitution R and M $\checkmark r = 5$ (2) $\checkmark$ substitution K and M $\checkmark r = 5$ (2)

Answer only 2/2

4.2.2	<p>T(-2 ; 0) TR = 8 units [line from centre <math>\perp</math> to chord]</p> <p><b>OR/OF</b></p> <p>M(2 ; 3) F(a ; 0) FR = 4 units TR = 8 units [line from centre <math>\perp</math> to chord]</p> <p><b>OR/OF</b></p> <p><math>(x-2)^2 + (0-3)^2 = 25</math>  <math>x^2 - 4x + 4 + 9 = 25</math>  <math>x^2 - 4x - 12 = 0</math>  <math>(x-6)(x+2) = 0</math>  <math>x = 6</math> or <math>x = -2</math>  TR = 8 units</p>	<p>✓ T(-2 ; 0) ✓ answer (2)</p> <p>✓ 4 units ✓ answer (2)</p> <p>✓ x values ✓ answer (2)</p>
4.3	<p><math>m_{\text{radius}} = \frac{7-3}{5-2}</math>  <math>m_{\text{radius}} = \frac{4}{3}</math>  <math>m_{\text{tangent}} = -\frac{3}{4}</math></p> <p><math>7 = -\frac{3}{4}(5) + c</math> <b>OR/OF</b> <math>y - 7 = -\frac{3}{4}(x - 5)</math></p> <p><math>c = \frac{43}{4}</math>  <math>y = -\frac{3}{4}x + \frac{43}{4}</math></p>	<p>✓ substitution  ✓ <math>m_{\text{radius}} = \frac{4}{3}</math>  ✓ <math>m_{\text{tangent}} = -\frac{3}{4}</math>  ✓ substitution  ✓ answer (5)</p>
4.4.1	N(2 ; -2)	<p>✓ <math>x_N = 2</math> ✓ <math>y_N = -2</math> (2)</p>
4.4.2	<p><math>(-2-2)^2 + (0+2)^2 = r^2</math>  <math>r^2 = 20</math>  <math>(x-2)^2 + (y+2)^2 = 20</math></p>	<p>✓ substitution  ✓ <math>r^2 = 20</math>  ✓ answer (3)</p>
		<b>[20]</b>

**Books and Journals**

- [1] B. Debar, *A Book of Values* (2012).
- [2] C. Olier and B. Debar, "Mining interesting articles", in: *The Journal of Software* (Jan. 1, 2008).

**Electronic Resources**

- [3] A. Author, *Using webkit*. Url: <http://www.example.org> (visited on 01/01/2013).

## BIBLIOGRAPHY

1	May/June 2022 – 2024 Marking Guidelines
2	